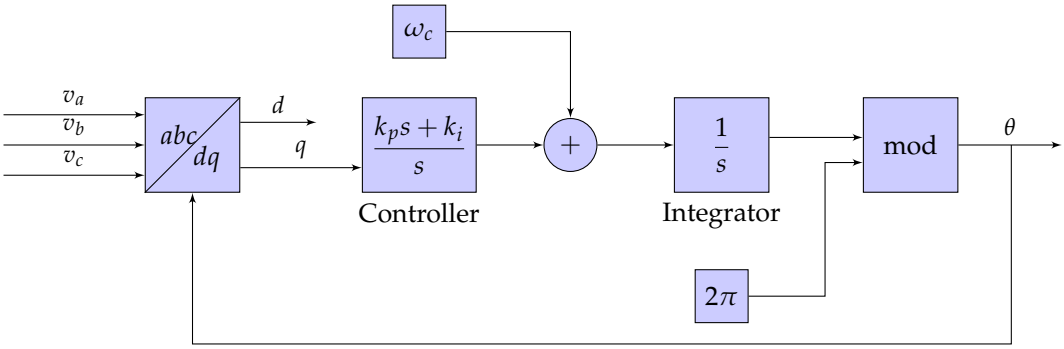
DIGITAL CONTROL OF POWER ELECTRONICS

Three Phase PLL Part 1



$$v_a = V_g \sin(\omega t) \tag{1}$$

$$v_b = V_g \sin(\omega t - 2\pi/3) \tag{2}$$

$$v_c = V_g \sin(\omega t - 4\pi/3) \tag{3}$$

The abc to dq transform is defined below

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ -\sin(\theta) & -\sin(\theta - 2\pi/3) & -\sin(\theta + 2\pi/3) \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{4}$$

Assume the input voltages are as follows
The controller is designed to drive the q component to zero, thus aligning the phase θ with the peak of the phase a voltage. The controller is a PI controller shown below

$$G_c(s) = \frac{k_p s + k_i}{s} \tag{5}$$

The output of the controller is the actuation signal and it is added to the center frequency ω_c. The output of the sum block is the frequency in radians and is integrated to obtain the angle θ.
The output of the integrator and the constant 2π is passed to a modulo operation. The output of the mod block properly wraps the angle θ and maintains it between 0 and 2π.
The open loop transfer function is

$$G_{OL}(s) = V_g \frac{k_p s + k_i}{s} \frac{1}{s} \tag{6}$$

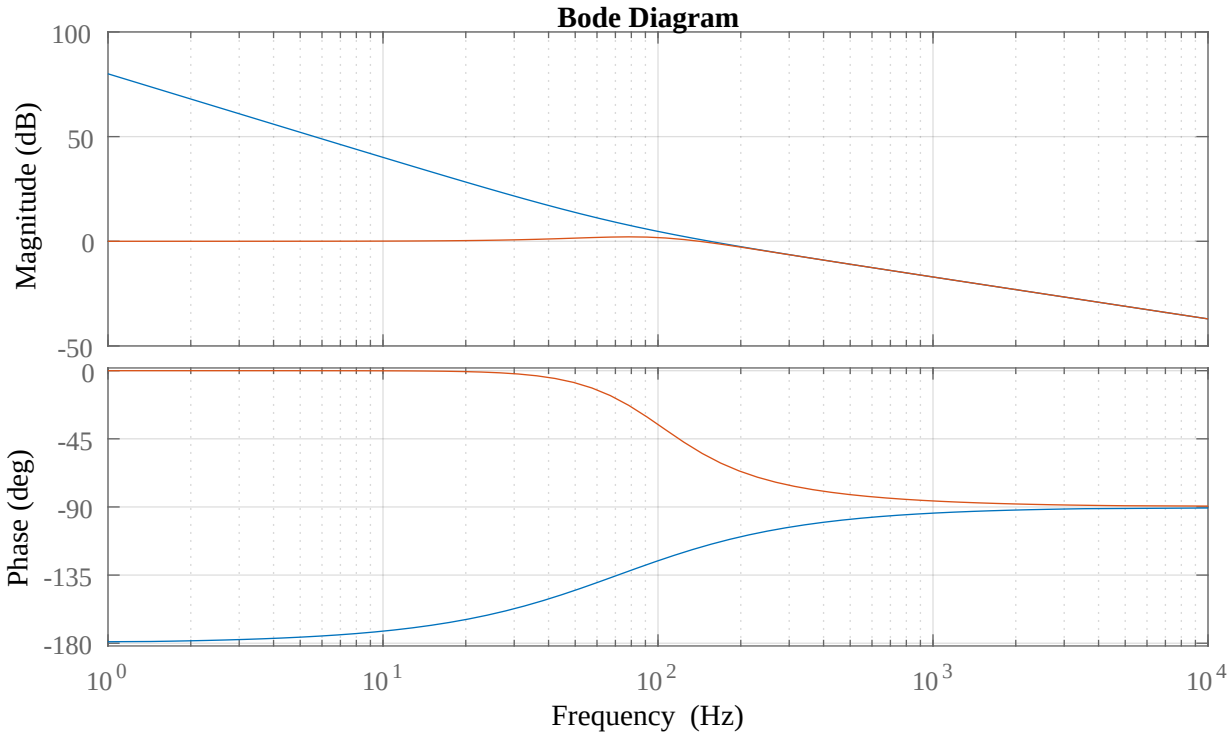
$$G_{CL}(s) = \frac{V_g(k_p s + k_i)}{s^2 + V_g k_p s + V_g k_i} \tag{7}$$

$$\omega_n = \sqrt{V_g k_i} \tag{8}$$

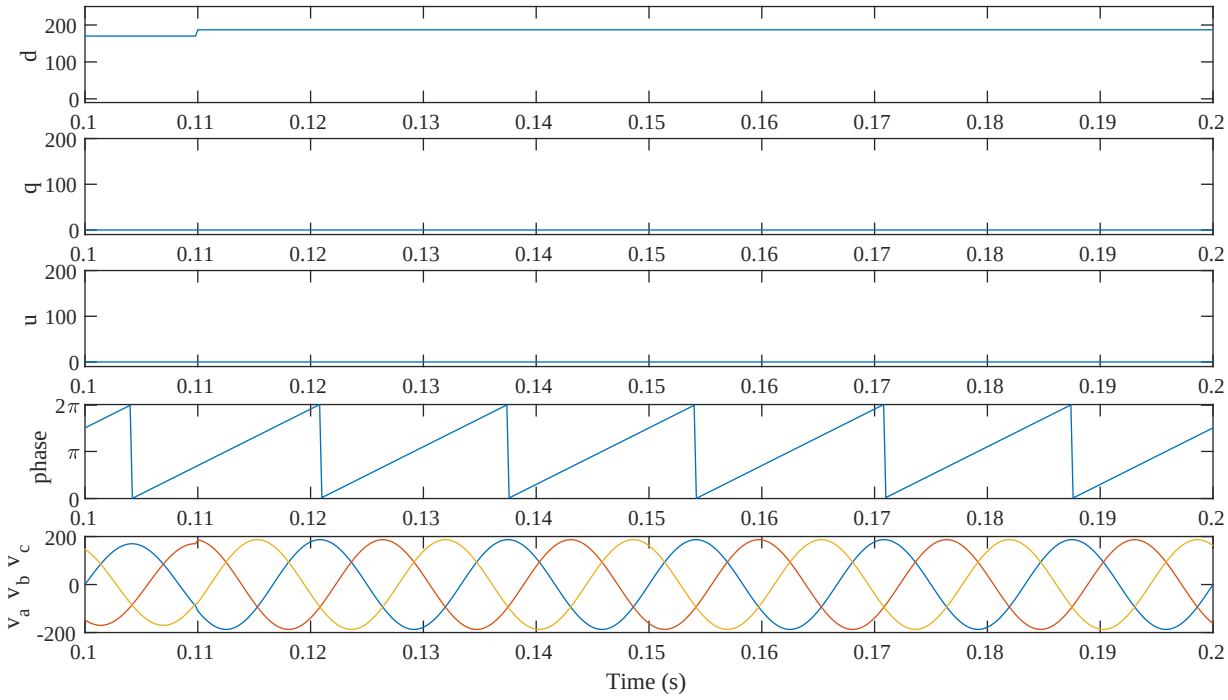
$$\zeta = \frac{V_g k_p}{2\sqrt{V_g k_i}} \tag{9}$$

Lets design the control loop for a damping ratio of $\zeta = 0.7$ and natural frequency of 100Hz assuming the grid voltage is 170.

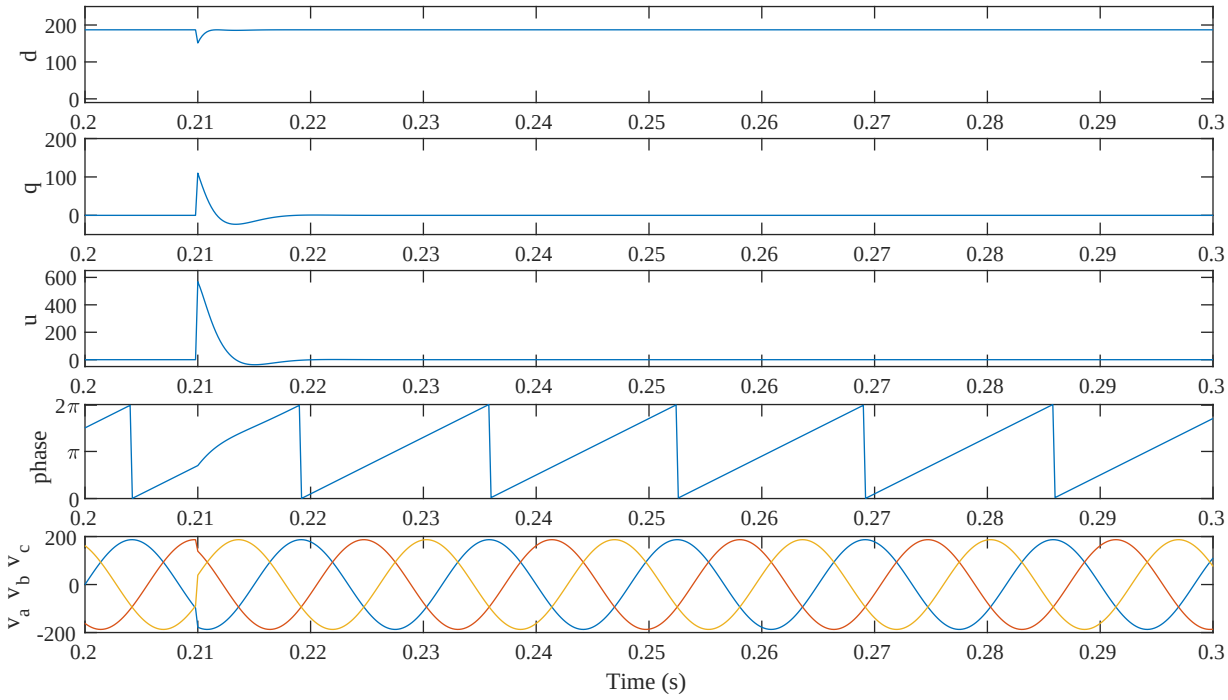
$$k_i = 2322.3 \quad k_p = 5.1744 \tag{10}$$


Bode Diagram
Magnitude (dB)
100
50
0
-50
Phase (deg)
0
-45
-90
-135
-180
10^0
10^1
10^2
10^3
10^4
Frequency (Hz)

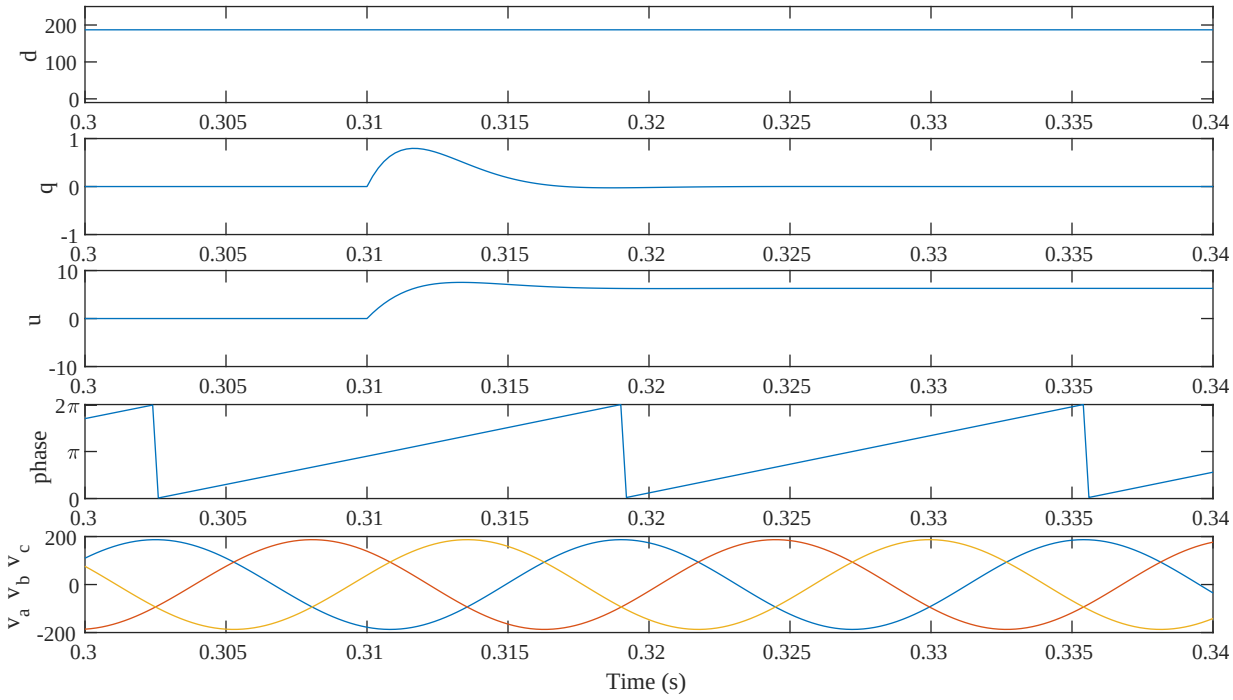
Show below is the d component, q component, control effort u, phase, and the line voltages in response to a voltage step of 17 volts at time 0.11.



Show below is the d component, q component, control effort u, phase, and the line voltages in response to a phase step of 0.6283 radians at time 0.21.



Show below is the d component, q component, control effort u, phase, and the line voltages in response to a frequency step of 1Hz at time 0.31.



Show below is the d component, q component, control effort u, phase, and the line voltages in response to a 10% voltage unbalance step at time 0.35.