DIGITAL CONTROL OF POWER ELECTRONICS

Proportional Integral Derivative Controller

The proportional integral derivate controller is shown below.

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s \tag{1}$$

The controller has three gains, K_i, K_p, K_d, that have to be designed.

$$G_c(s) = \frac{K_p\left(s^2 + \frac{K_p}{K_d}s + \frac{K_i}{K_d}\right)}{s} \tag{2}$$

Note that a PID controller has two zeros that can be placed, and gain, and a pole at the origin.
The zeros of the controller are

$$r_{1,2} = -\frac{K_p}{2K_d} \pm \sqrt{\left(\frac{K_p}{2K_d}\right)^2 - \frac{K_i}{K_d}} \tag{3}$$

The zeros fix two ratios of gains. The third variable then is the selection of K_d.
Lets do an example with the following plant

$$G_p(s) = \frac{25}{s^2 + 4s + 3} \tag{4}$$

The plant has two poles at -3 and -1. Lets cancel both poles with the PID controller.

$$\frac{K_p}{K_d} = 4, \quad \frac{K_i}{K_d} = 3 \tag{5}$$

$$G_{ol}(s) = \frac{25}{s^2 + 4s + 3}\frac{K_p\left(s^2 + \frac{K_p}{K_d}s + \frac{K_i}{K_d}\right)}{s} = \frac{K_p 25}{s} \tag{6}$$

After the pole cancellation, you can adjust the bandwidth by the selection of K_p. Lets find the gain for a bandwidth of 100Hz.

$$K_p = \frac{2\pi 100}{25} = 8\pi \quad K_i = 75.3982 \quad K_p = 100.531 \tag{7}$$

The resulting PID controller is

$$G_c(s) = \frac{25.13s^2 + 100.5s + 75.4}{s} \tag{8}$$

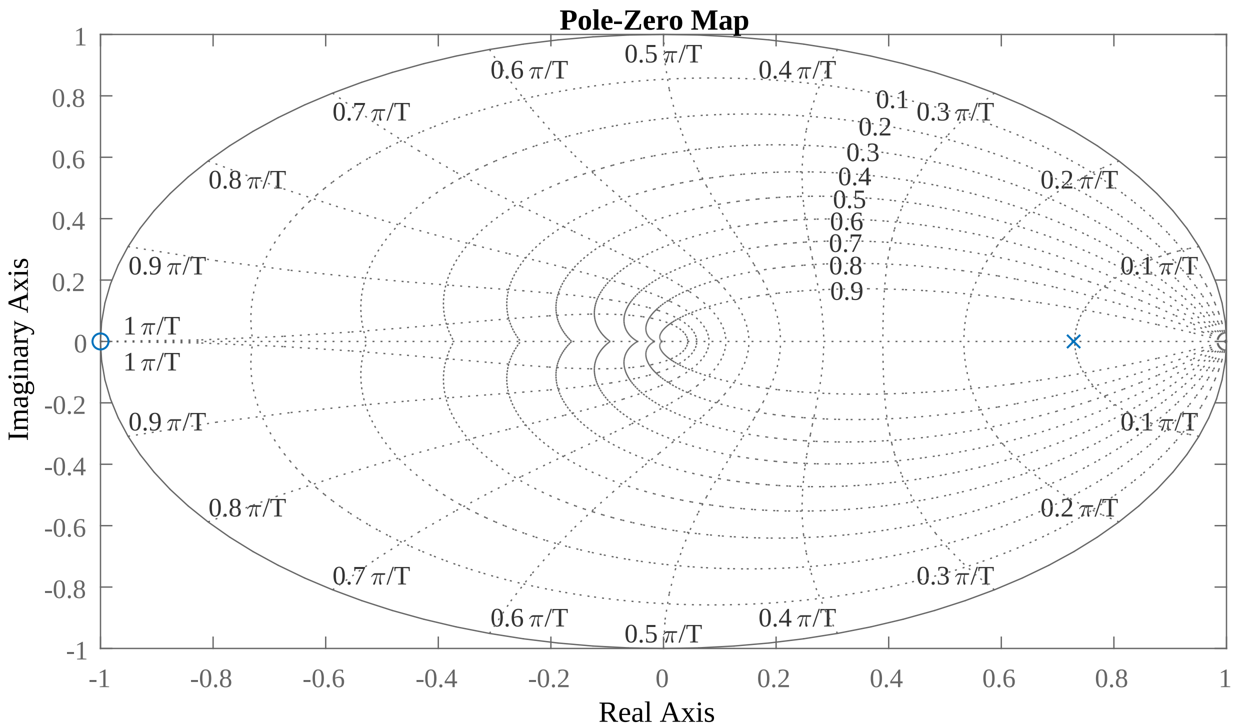
Now we need to convert this to a digital controller using tustin transformation and $T = 0.001$

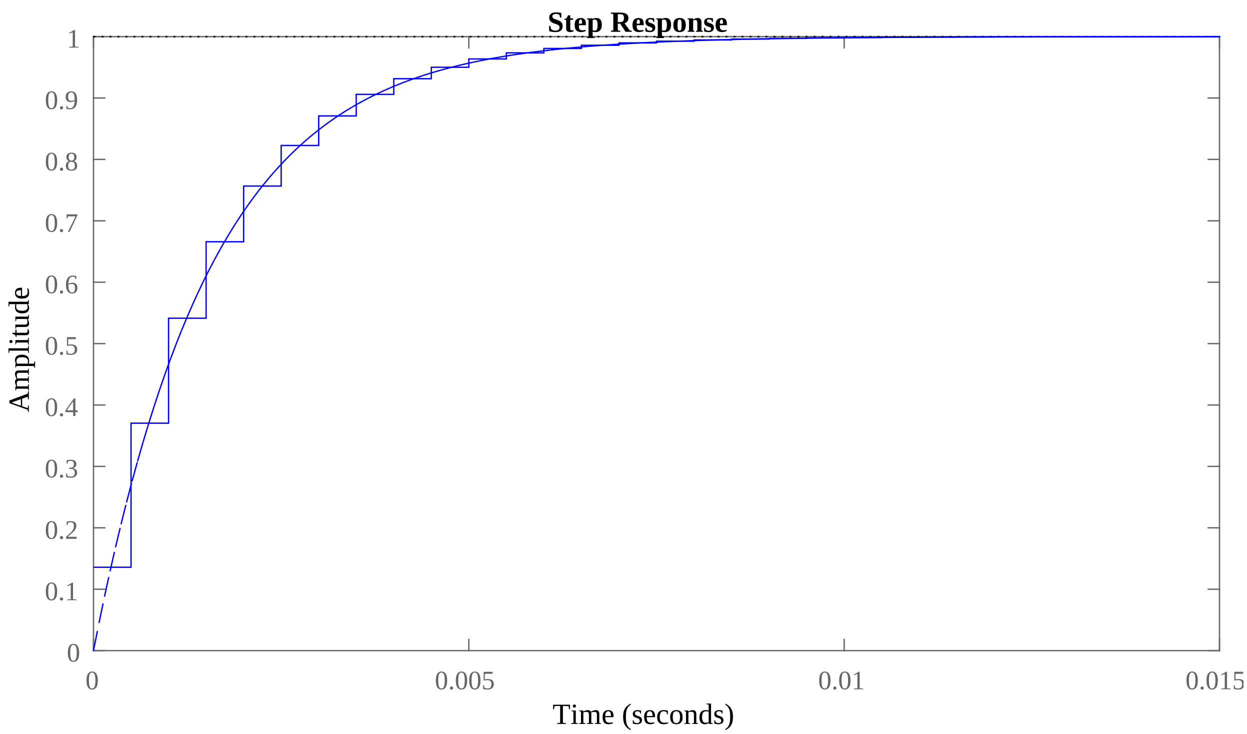
$$G_c(z) = \frac{100631z^2 - 201061z + 100430}{z^2 - 1} \tag{9}$$

The closed loop system transfer function in continuous time is shown below.

$$G_{cl}(s) = \frac{628.3}{s + 628.3} \tag{10}$$

$$G_{cl}(z) = \frac{0.1358z + 0.1358}{z - 0.7285} \tag{11}$$

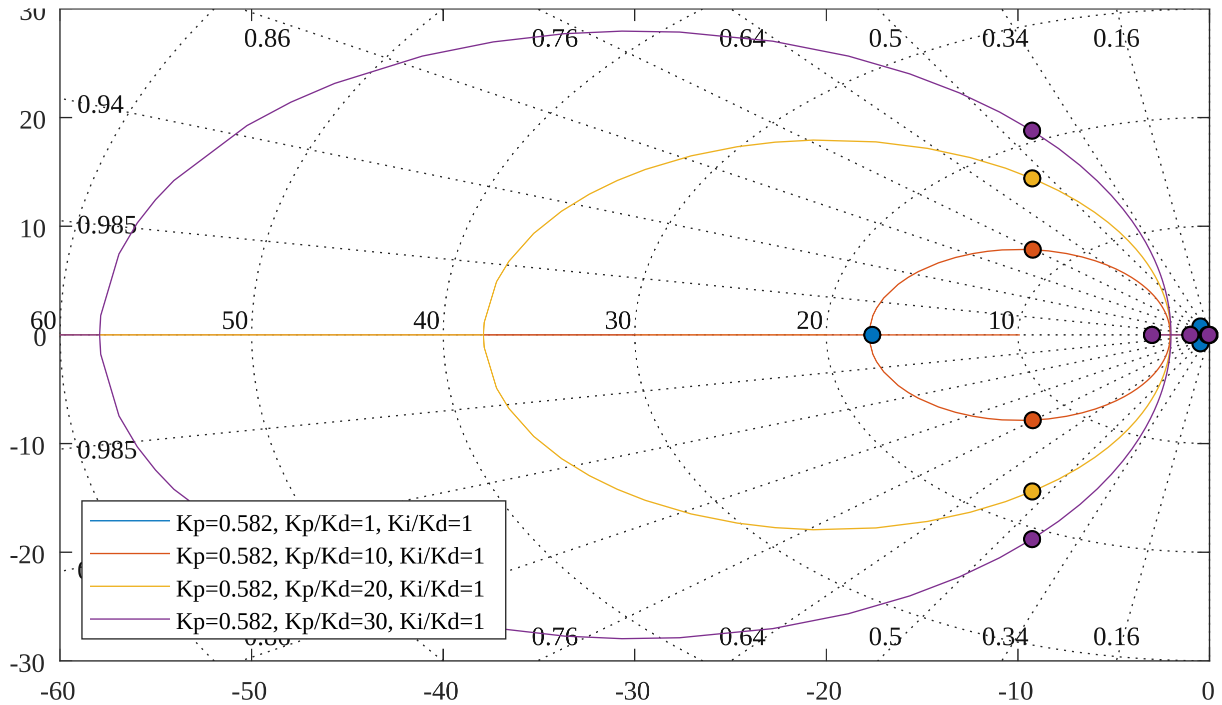




Another approach to designing a PID controller is to explore all of the gain space for the PID controller. A useful way to do this is through the root locus approach. The root locus approach will plot the poles of the system as the gains are varied.

The gain K_p of the open loop transfer function below is swept for fixed ratios of $\frac{K_p}{K_d}$ and $\frac{K_i}{K_d}$

$$G_c(s) = \frac{K_p \left(s^2 + \frac{K_p}{K_d} s + \frac{K_i}{K_d} \right)}{s} \tag{12}$$

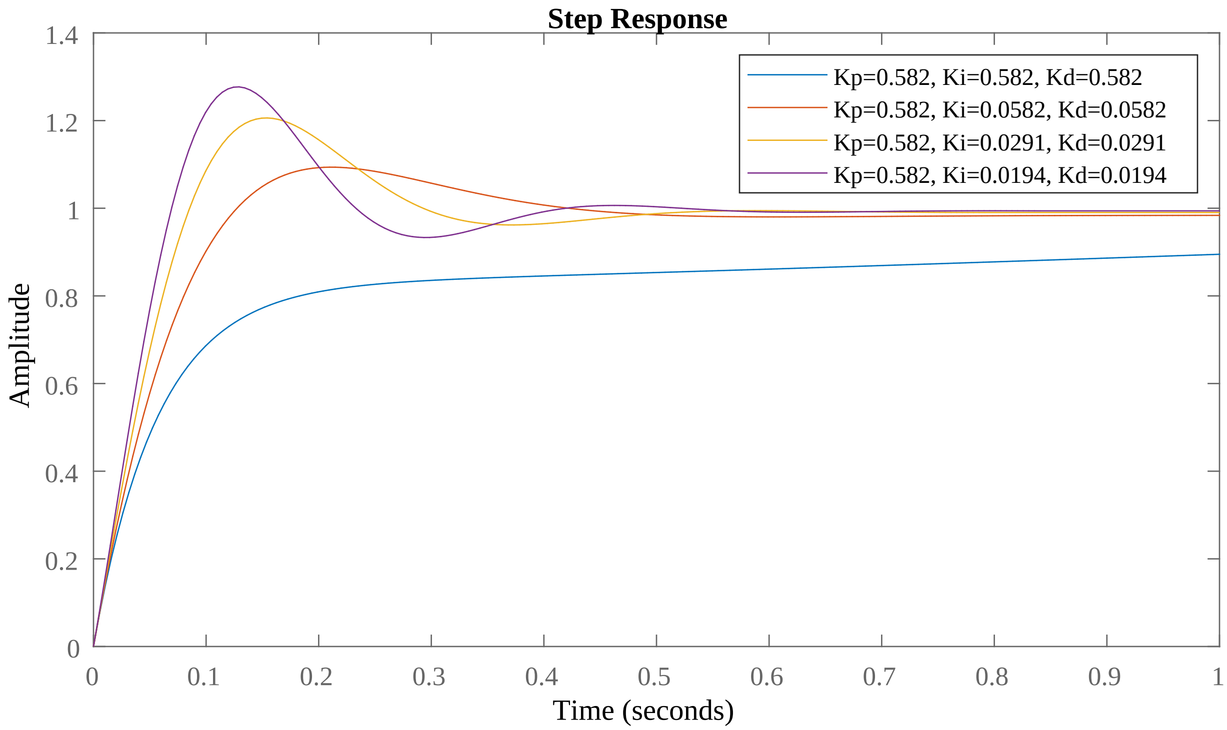


$$G_c(s) = \frac{291\ \left(s^2 + s + 1\right)}{500\, s} \tag{13}$$

$$G_c(s) = \frac{291\ \left(s^2 + 10\, s + 1\right)}{500\, s} \tag{14}$$

$$G_c(s) = \frac{291\ \left(s^2 + 20\, s + 1\right)}{500\, s} \tag{15}$$

$$G_c(s) = \frac{291\ \left(s^2 + 30\, s + 1\right)}{500\, s} \tag{16}$$

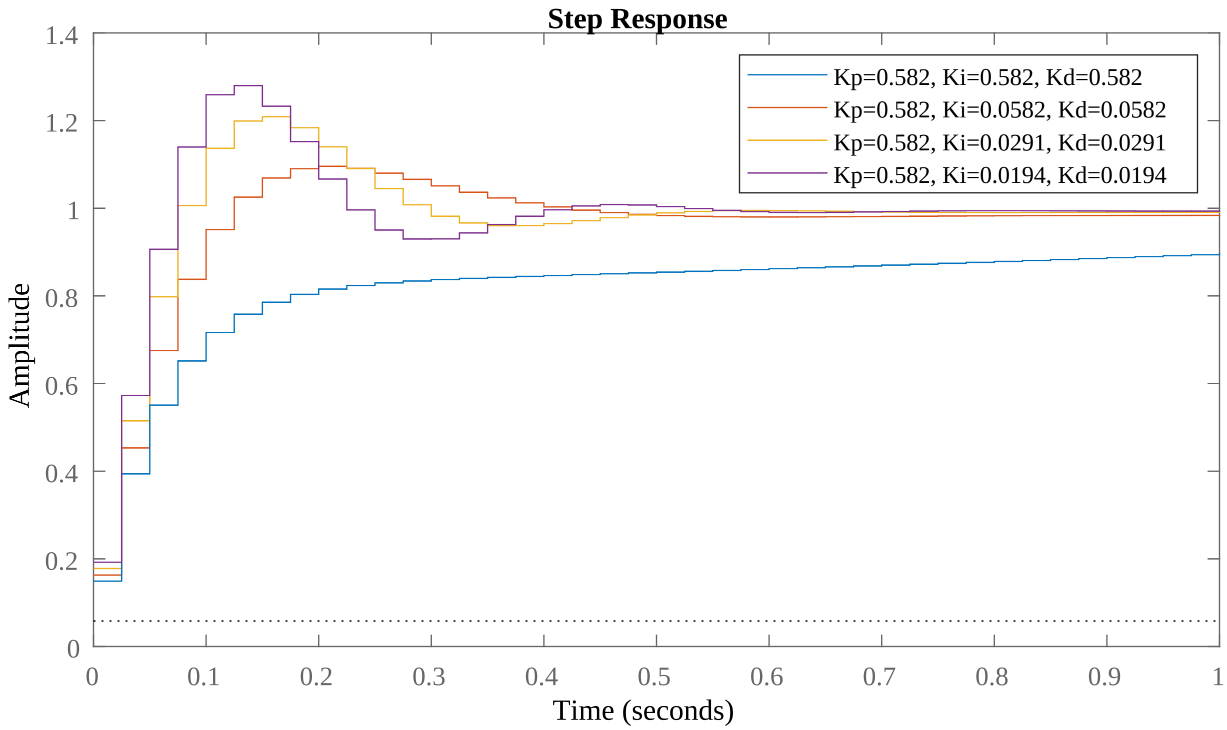


$$G_c(z) = \frac{47.149\,z^2 - 93.105\,z + 45.985}{z^2 - 1.0} \tag{17}$$

$$G_c(z) = \frac{52.387\,z^2 - 93.105\,z + 40.747}{z^2 - 1.0} \tag{18}$$

$$G_c(z) = \frac{58.207\,z^2 - 93.105\,z + 34.927}{z^2 - 1.0} \tag{19}$$

$$G_c(z) = \frac{64.027\,z^2 - 93.105\,z + 29.107}{z^2 - 1.0} \tag{20}$$


Step Response
Kp=0.582, Ki=0.582, Kd=0.582
Kp=0.582, Ki=0.0582, Kd=0.0582
Kp=0.582, Ki=0.0291, Kd=0.0291
Kp=0.582, Ki=0.0194, Kd=0.0194
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1