DIGITAL CONTROL OF POWER ELECTRONICS

Proportional Integral Controller

A continuous time PI controller is shown below.

$$G_c(s) = K_p + \frac{K_i}{s} \tag{1}$$

Lets now use the tustin transformation

$$s = \frac{2(z-1)}{T(z+1)} \tag{2}$$

to convert the PI controller to the digital domain.

$$G_c(z) = \frac{z(2K_p + K_iT) - 2K_p + K_iT}{2(z-1)} \tag{3}$$

Note that there is one zero and one pole introduced by the digital PI controller.
Lets design a PI controller in the continuous time domain for the plant below

$$G_p = \frac{10}{(s+7)} \tag{4}$$

The PI controller

$$G_c(s) = \frac{K_p(s + K_i/K_p)}{s} \tag{5}$$

Lets set the zero of the PI controller to cancel the pole of the plant

$$G_{ol} = \frac{10}{(s+7)} \frac{K_p(s + K_i/K_p)}{s} \tag{6}$$

Let the ratio be

$$\frac{K_i}{K_p} = 7 \tag{7}$$

We have two variables K_i and K_p. The ratio between the two is now fixed which leaves us with one degree of freedom.
Now how do i now select the gains? Lets set the bandwidth!

$$G_{ol} = \frac{10K_p}{s} \tag{8}$$

Set the proportional gain to be

$$K_p = \frac{2\pi f_{band}}{10} \tag{9}$$

Lets do bandwidth of 100Hz.

$$K_i = 62.8319 \quad K_p = 439.8230 \tag{10}$$

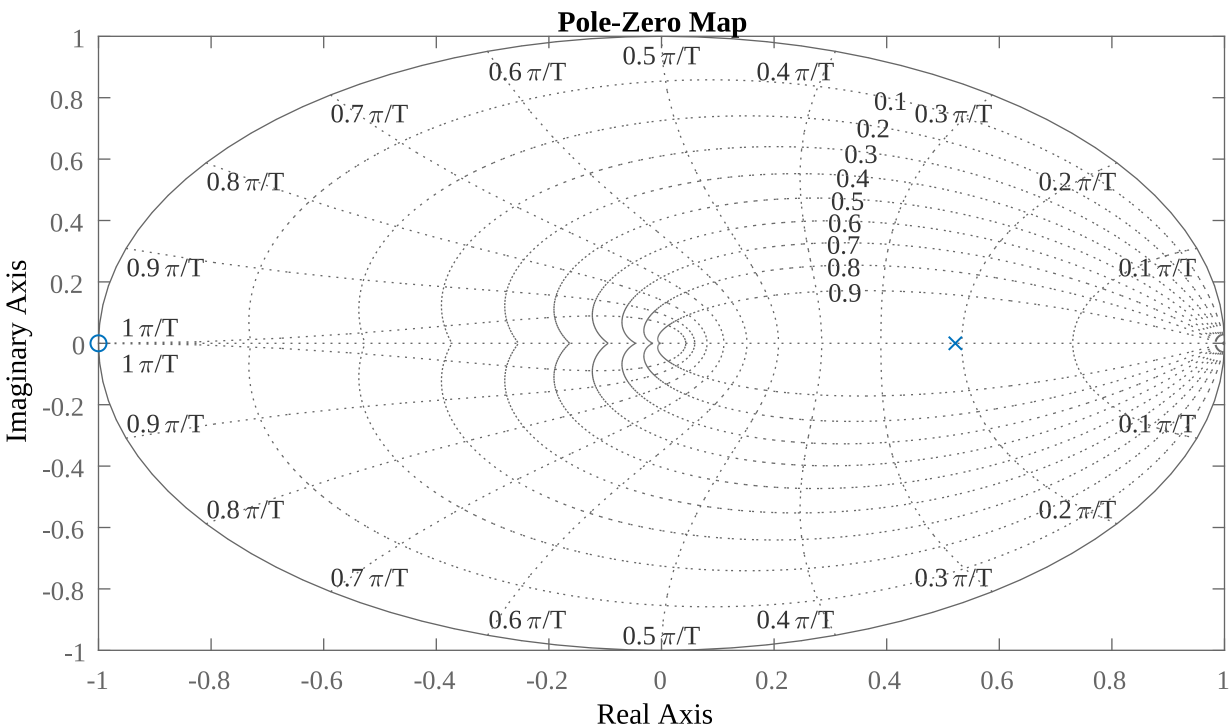
$$G_c(s) \frac{62.83s + 439.8}{s} \tag{11}$$

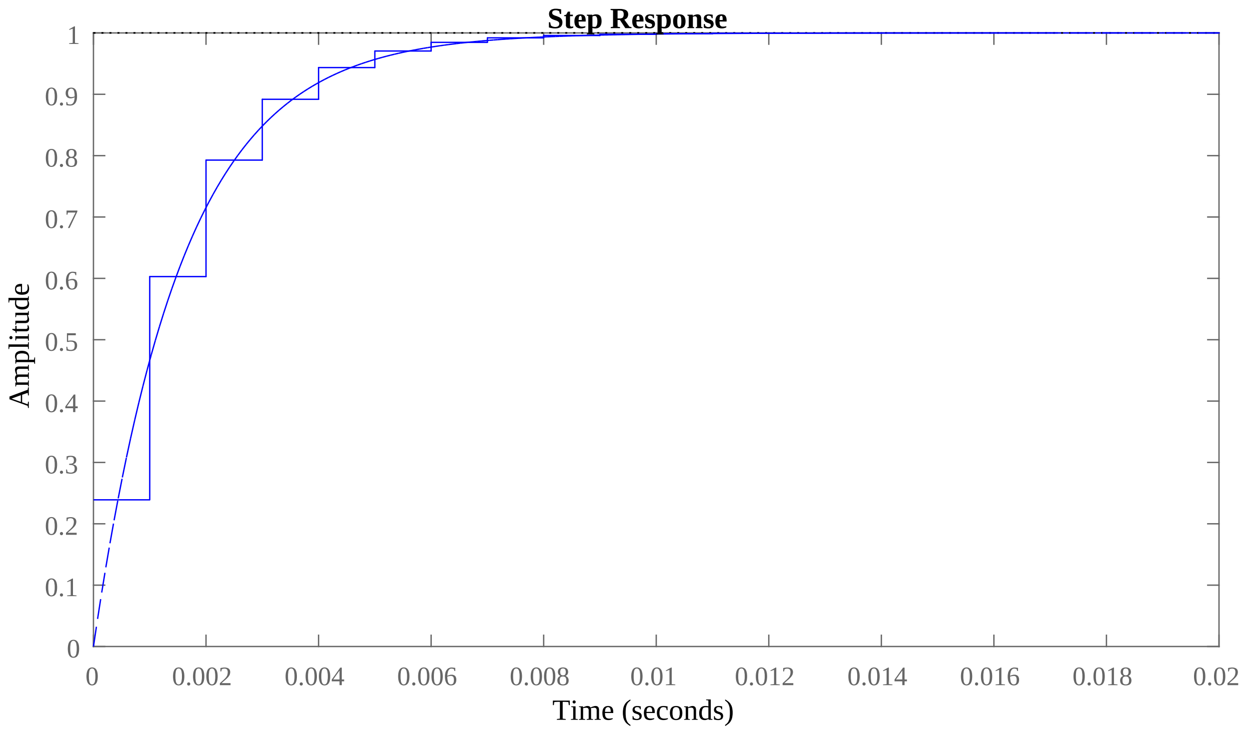
$$G_c(z) = \frac{63.05z - 62.61}{z-1} \tag{12}$$

$$G_{cl}(s) = \frac{628.3}{s + 628.3} \tag{13}$$

$$G_{cl}(z) = \frac{0.2391z + 0.2391}{z - 0.5219} \tag{14}$$

There is only one pole at its located at 0.5219. The system is stable and should exhibit a damped exponential response.

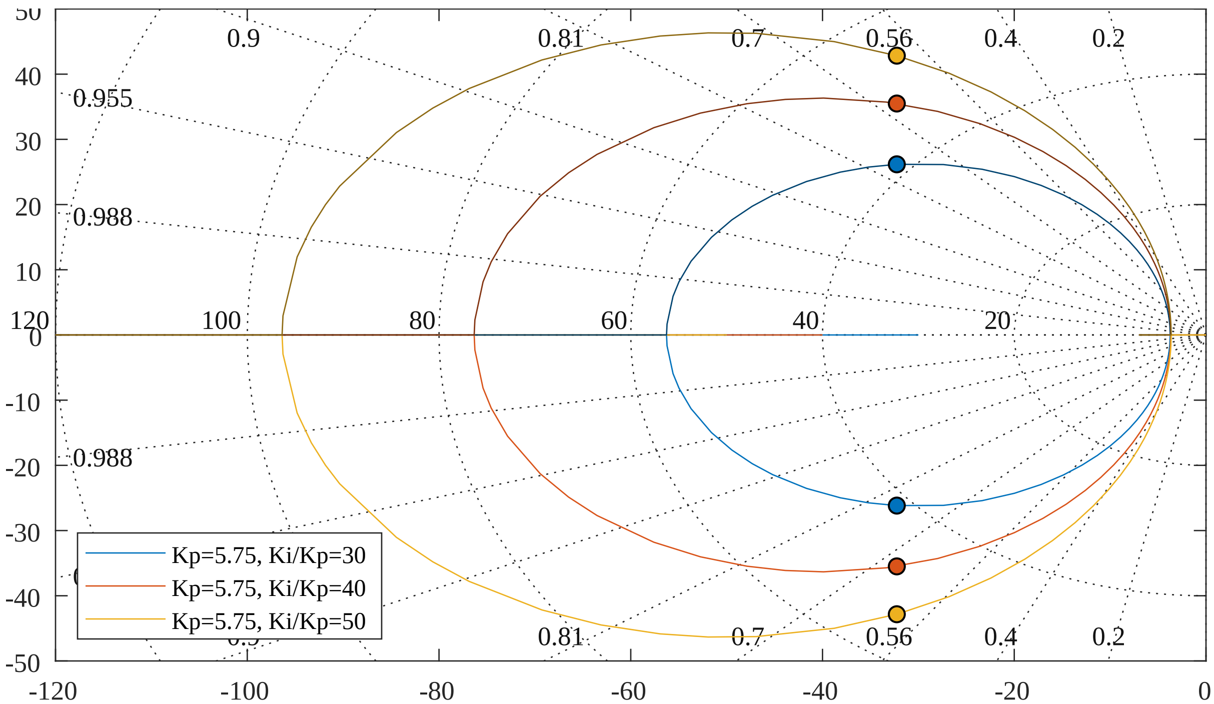


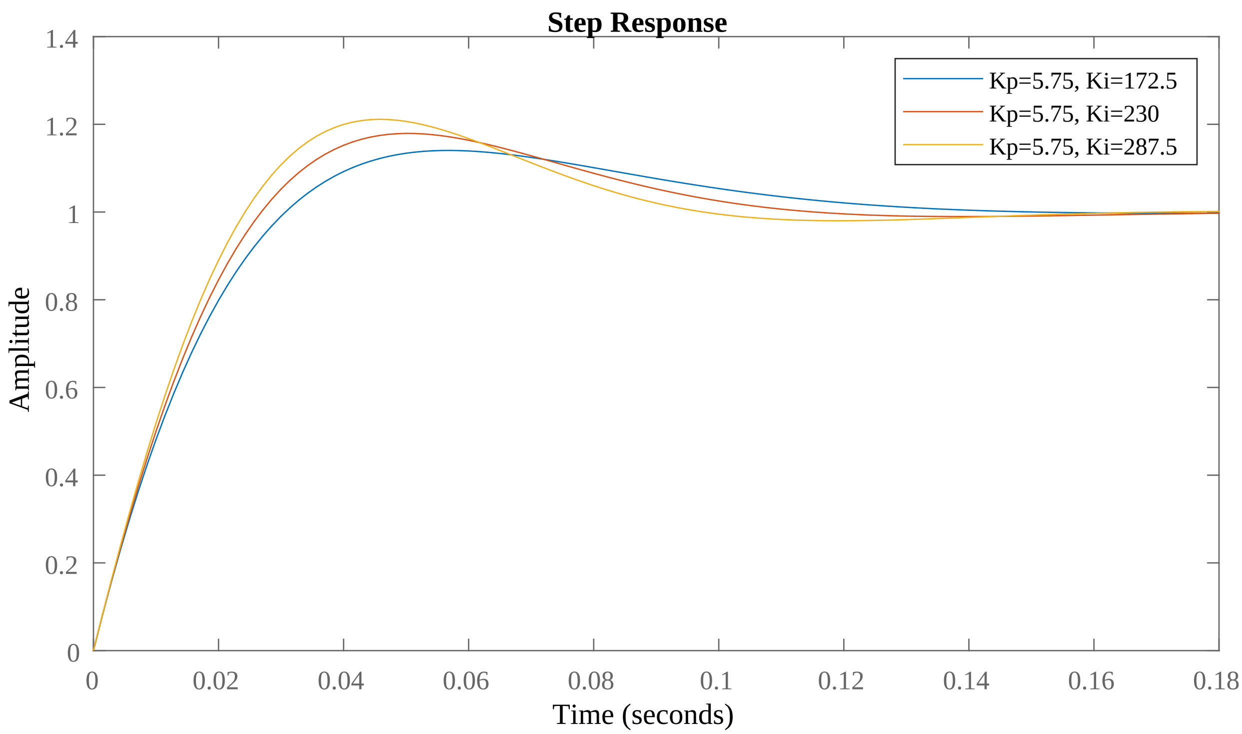


Another approach to designing a PI controller is to explore all of the gain space for the PI controller. A useful way to do this is through the root locus approach. The root locus approach will plot the poles of the system as the gains are varied.

The gain K_i of the open loop transfer function below is swept for fixed ratios of $\frac{K_i}{K_p}$

$$G_{ol} = \frac{10}{(s+7)} \frac{K_p(s + K_i/K_p)}{s} \tag{15}$$





$$T_s = 0.01 \tag{16}$$

$$G_c(z) = \frac{6.612z - 4.887}{z - 1} \tag{17}$$

$$G_c(z) = \frac{6.9z - 4.6}{z - 1} \tag{18}$$

$$G_c(z) = \frac{7.188z - 4.313}{z - 1} \tag{19}$$