DIGITAL CONTROL OF POWER ELECTRONICS

Intro to Digital Control Part 3

Z-transform

The definition of the z-transform is given below

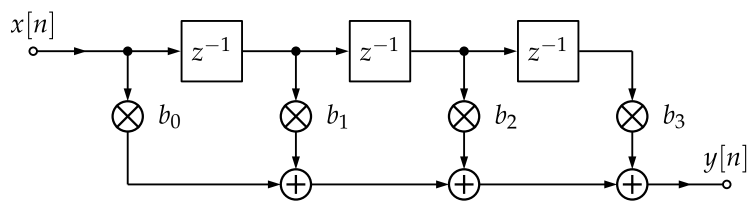
$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \tag{1}$$

The transfer function for a linear, time-invariant, digital filter can be expressed as a transfer function in the Z-domain shown below

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + ... + b_N z^{-N}}{1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_N z^{-N}} \tag{2}$$

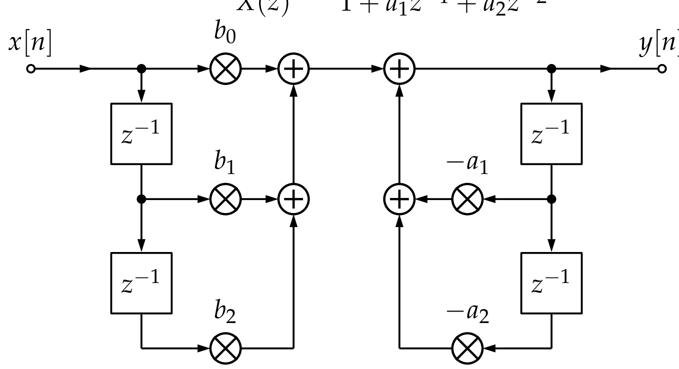
The pole zero plot is very helpful in determining stability, speed of response
Shown below is a Finite Impulse Response filter (FIR). The equation of that filter is shown here

$$H(z) = \frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} \tag{3}$$



Shown below is a Infinite Impulse Response filter (IIR). The equation of that filter is shown here

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{4}$$



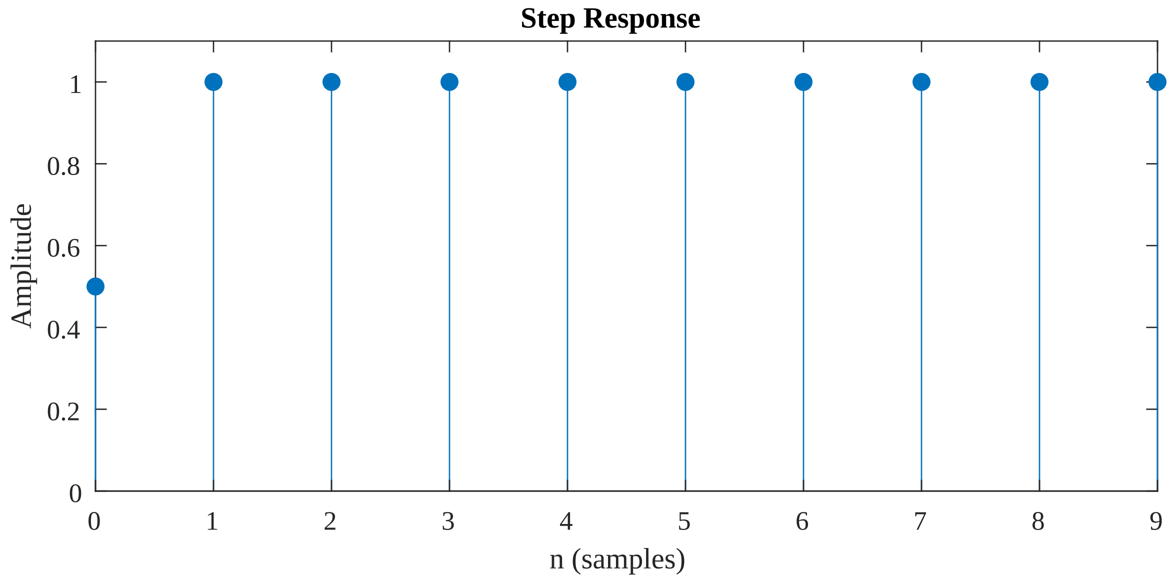
Consider the difference equation below.

$$y[n] = \frac{x[n] + x[n-1]}{2} \tag{5}$$

Convert the difference equation below to the z-domain.

$$\frac{Y(z)}{X(z)} = 0.5 + 0.5z^{-1} \tag{6}$$

The result is a simple 2 element FIR moving average filter. Below shown is the response of the filter to a step input.



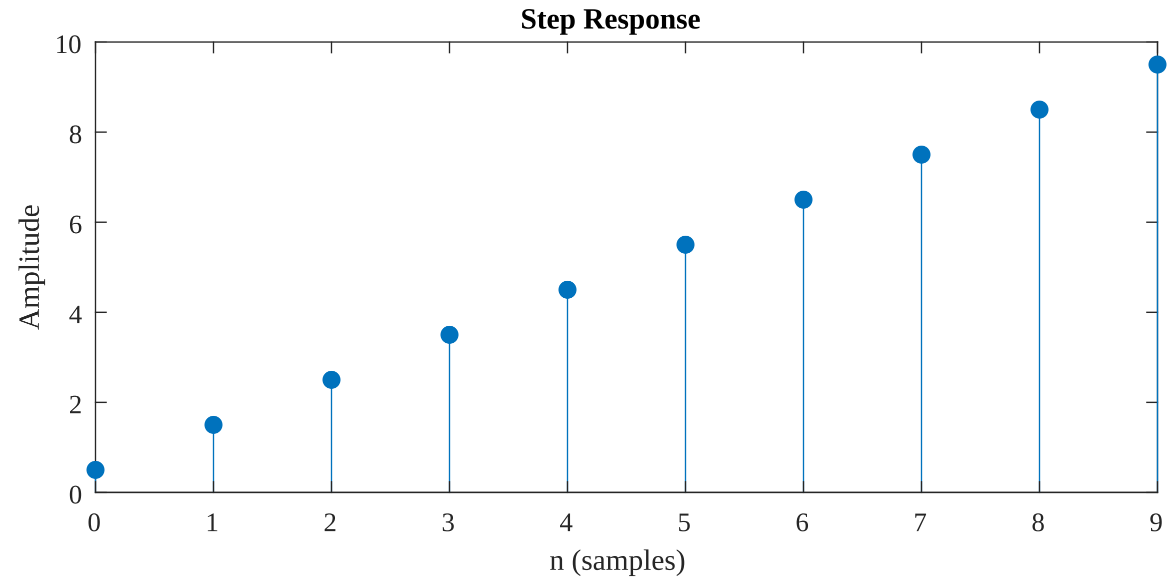
Consider the difference equation below for trapezoidal integration.

$$y[n] = y[n-1] + \frac{T}{2}(x[n] + x[n-1]) \tag{7}$$

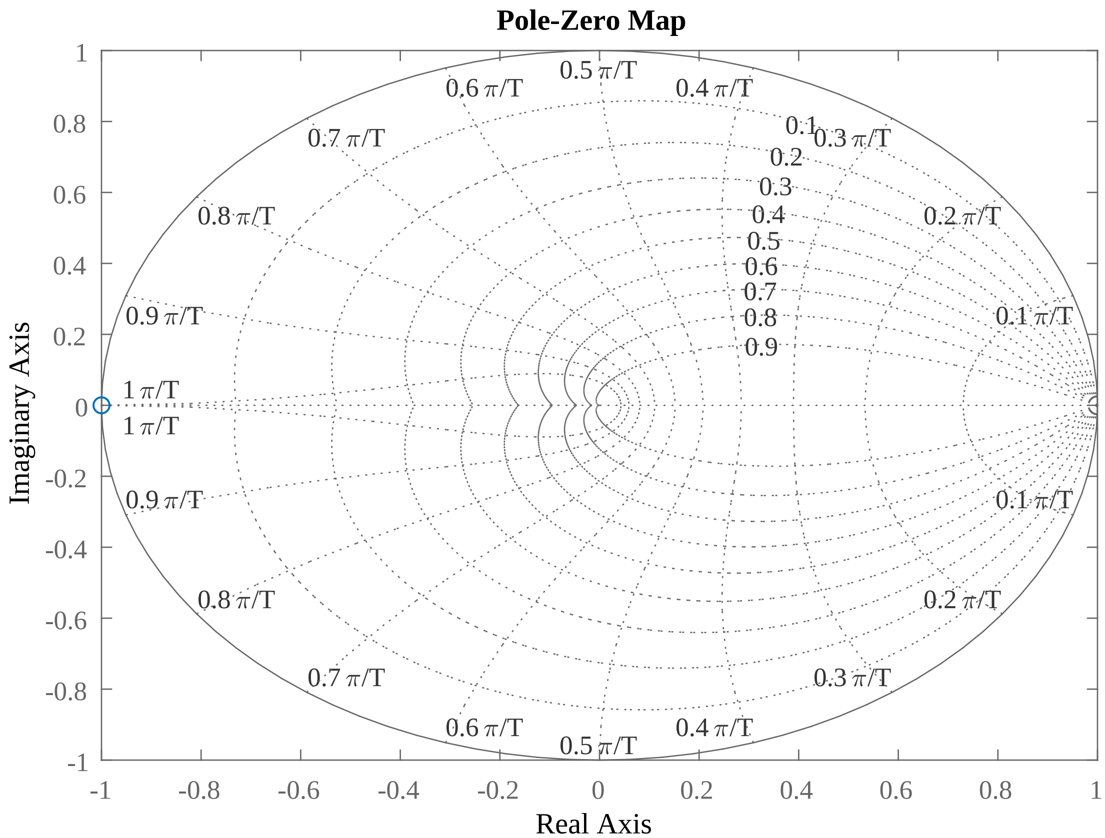
Convert the difference equation to the z-domain

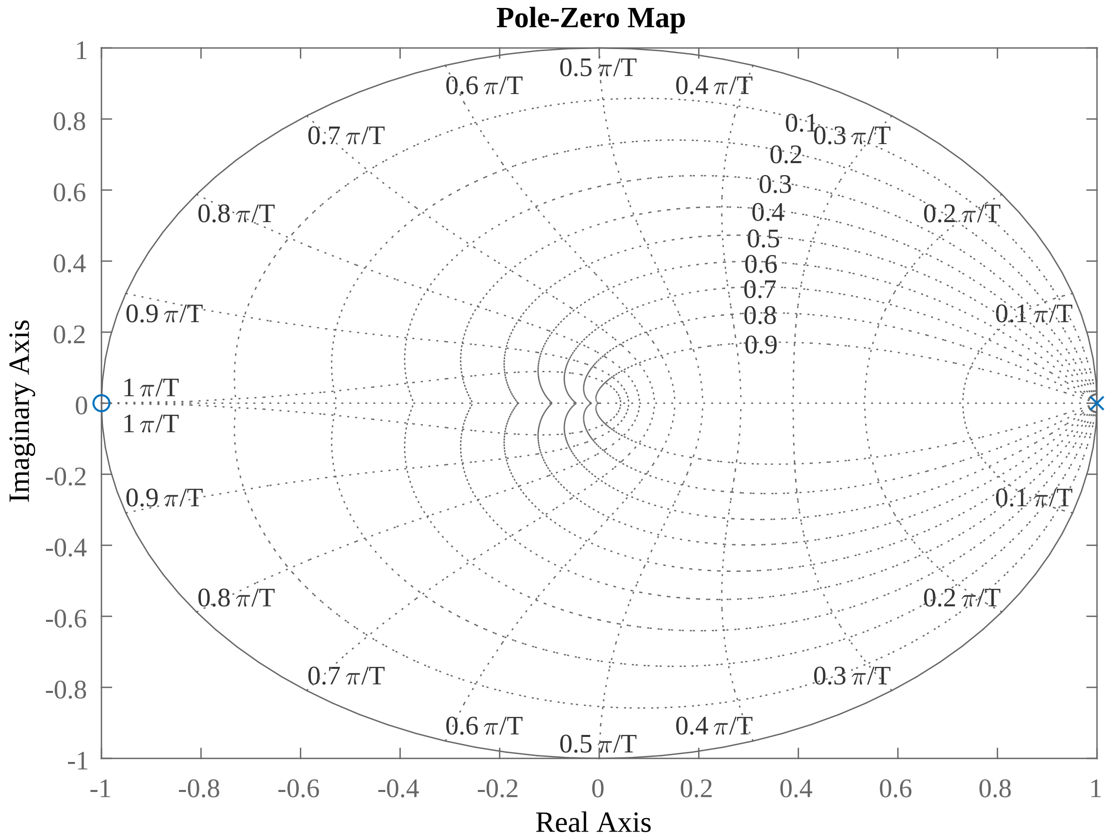
$$\frac{Y(z)}{X(z)} = \frac{T}{2}\frac{1+z^{-1}}{1-z^{-1}} \tag{8}$$

The response of the trapezoidal integrator to a step input is shown below.



Pole-Zero Map





Forward Euler

Forward integration difference equation yields

$$y[n+1] = x[n]T + y[n] \tag{9}$$

convert the difference equation to the z-domain using the z transform.

$$Y(z)z = X(z)T + Y(z) \tag{10}$$

Solve for transfer function from input to output

$$\frac{Y(z)}{X(z)} = \frac{T}{z-1} = \frac{Tz^{-1}}{1-z^{-1}} \tag{11}$$

The s in the laplace domain can be approximated by

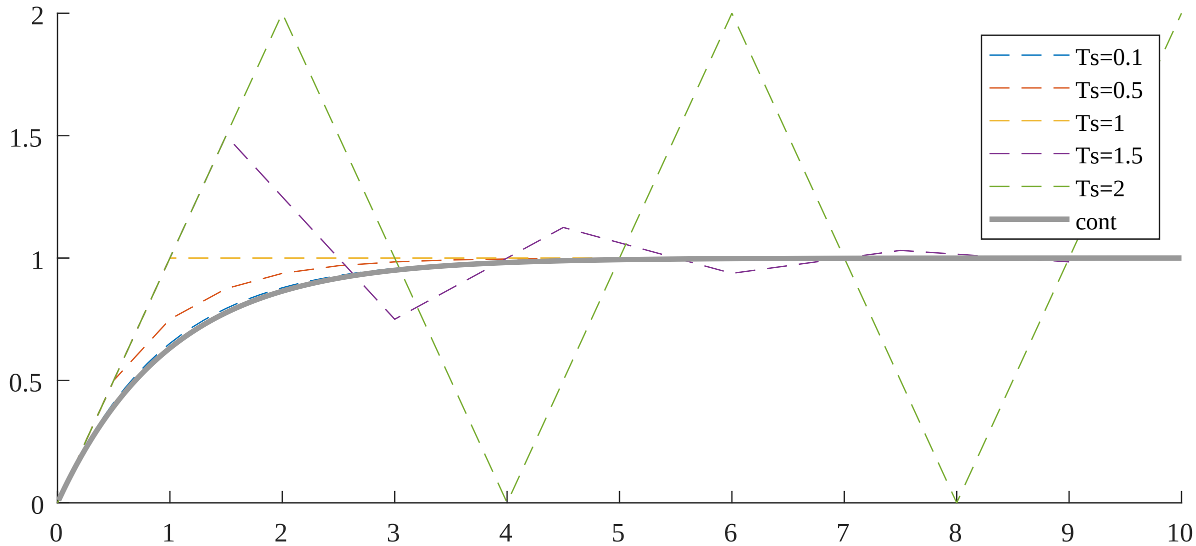
$$s \approx \frac{z-1}{T} = \frac{1-z^{-1}}{Tz^{-1}} \tag{12}$$

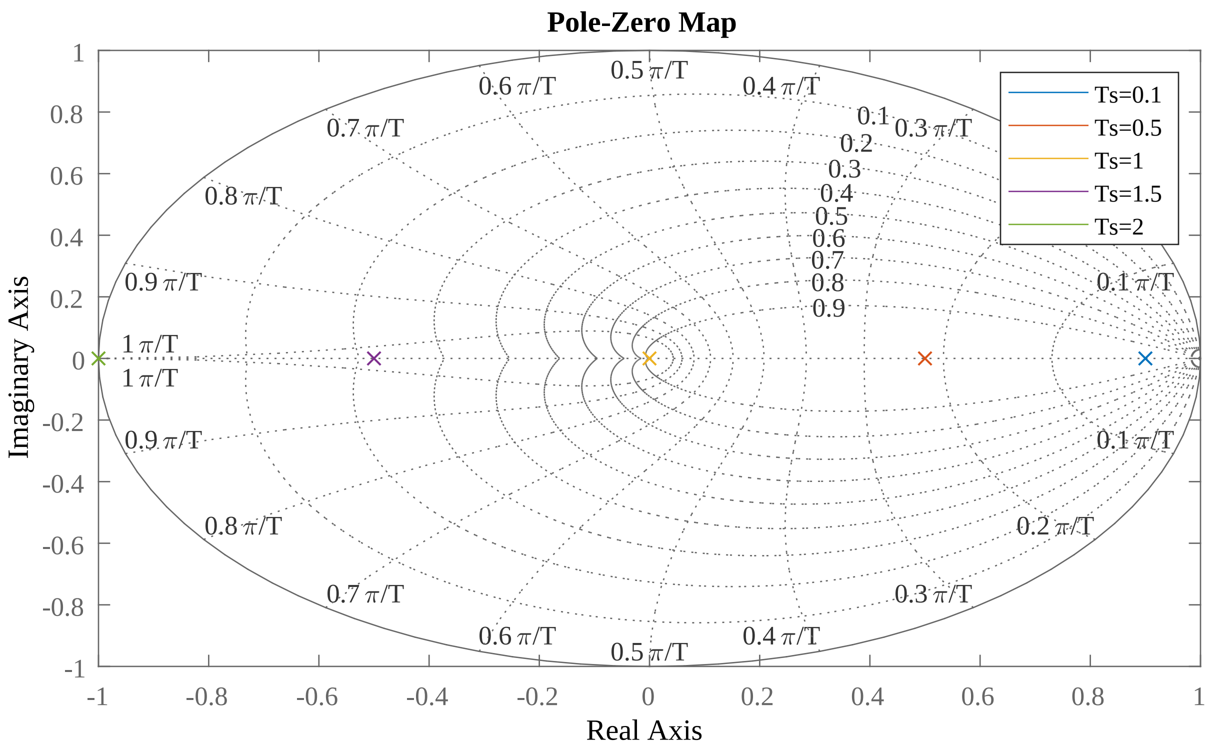
Lets consider the low pass filter below

$$G(s) = \frac{1}{s+1} \tag{13}$$

Convert the transfer function to the z-domain using forward euler approximation for s.

$$G(z) = \frac{Tz^{-1}}{1-(1-T)z^{-1}} \tag{14}$$





Backward Euler

Forward integration difference equation yields

$$y[n] = x[n]T + y[n-1] \tag{15}$$

convert the difference equation to the z-domain using the z transform.

$$Y(z) = X(z)T + Y(z)z^{-1} \tag{16}$$

Solve for transfer function from input to output

$$\frac{Y(z)}{X(z)} = \frac{Tz}{z-1} = \frac{T}{1-z^{-1}} \tag{17}$$

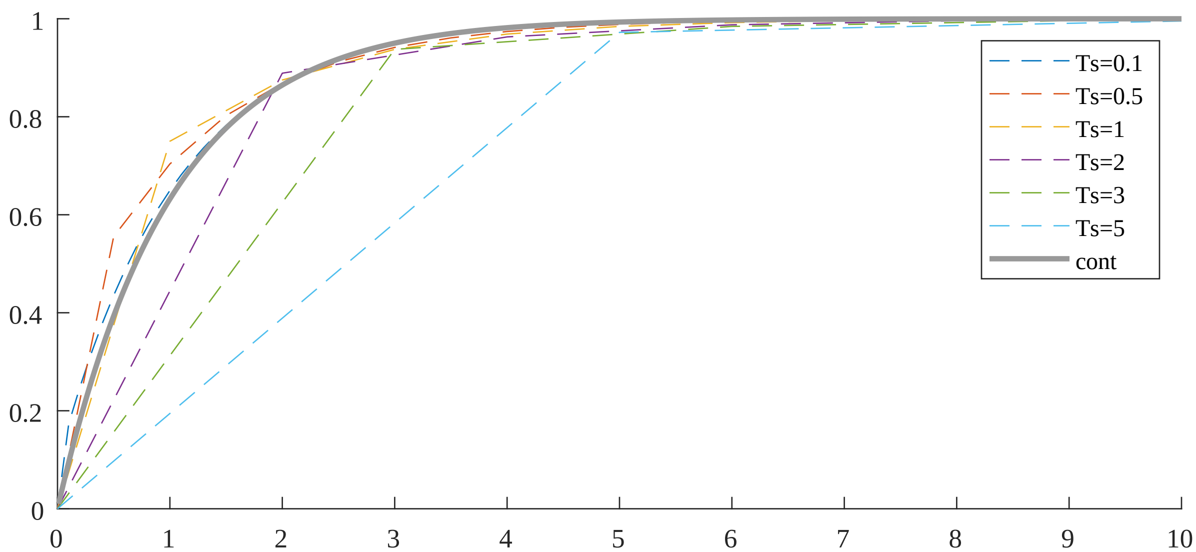
$$s \approx \frac{z-1}{Tz} = \frac{1-z^{-1}}{T} \tag{18}$$

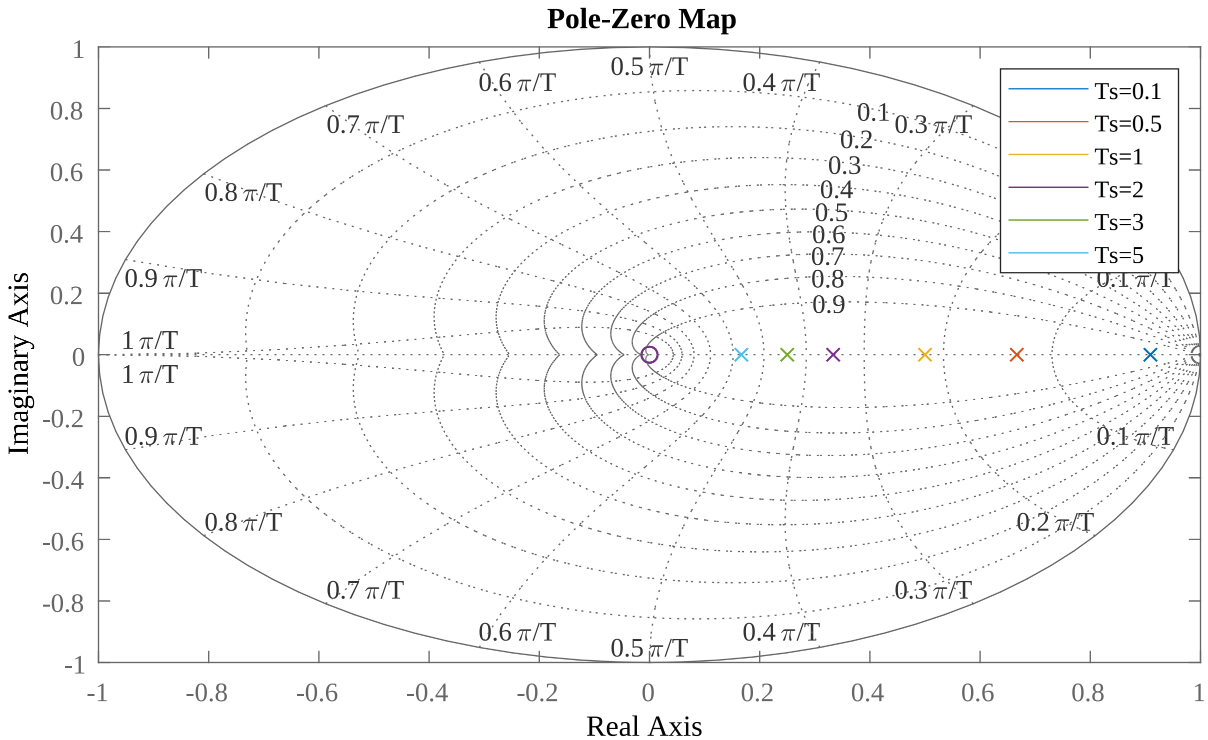
Lets consider the low pass filter below

$$G(s) = \frac{1}{s+1} \tag{19}$$

Convert the transfer function to the z-domain using forward euler approximation for s.

$$G(z) = \frac{T}{(1+T) - z^{-1}} \tag{20}$$





Trapezoidal Integration

Trapezoidal Integration

$$y(n) = (x(n) + x(n-1))\,\frac{T}{2} + y(n-1) \tag{21}$$

Take the z-transform

$$Y(z) - Y(z)z^{-1} = \frac{T}{2}\left(X(z) + X(z)z^{-1}\right) \tag{22}$$

$$Y(z)(1-z^{-1}) = \frac{T}{2}\left(1+z^{-1}\right)X(z) \tag{23}$$

$$\frac{Y(z)}{X(z)} = \frac{T}{2}\left(\frac{1+z^{-1}}{1-z^{-1}}\right) = \frac{T}{2}\left(\frac{z+1}{z-1}\right) \tag{24}$$

$$H(s) = \frac{1}{s} \tag{25}$$

The tustin approximation or bilinear transform is

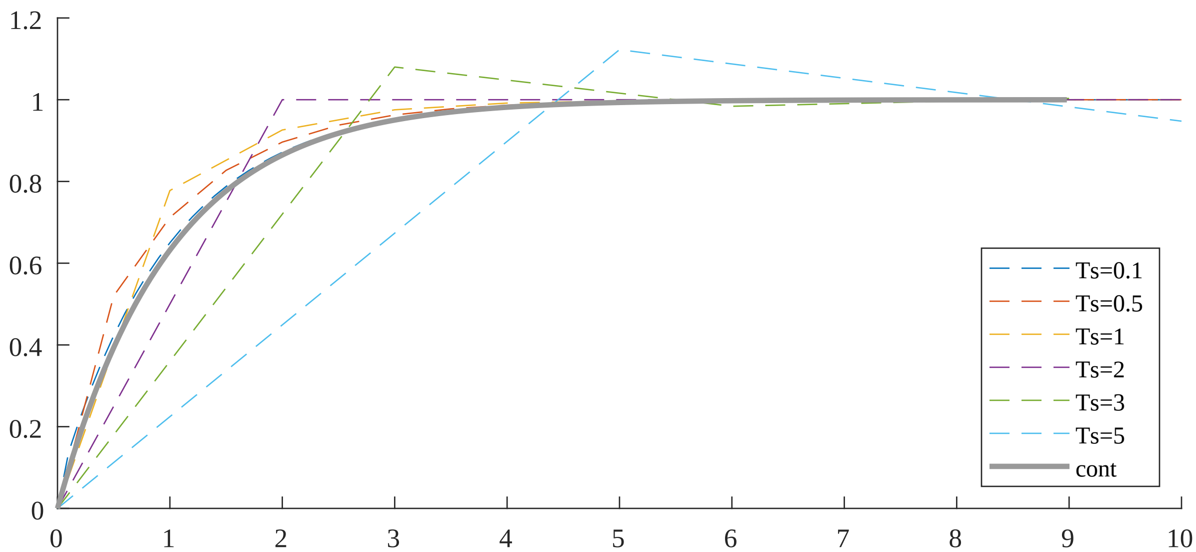
$$s \approx \frac{2}{T}\frac{z-1}{z+1} \tag{26}$$

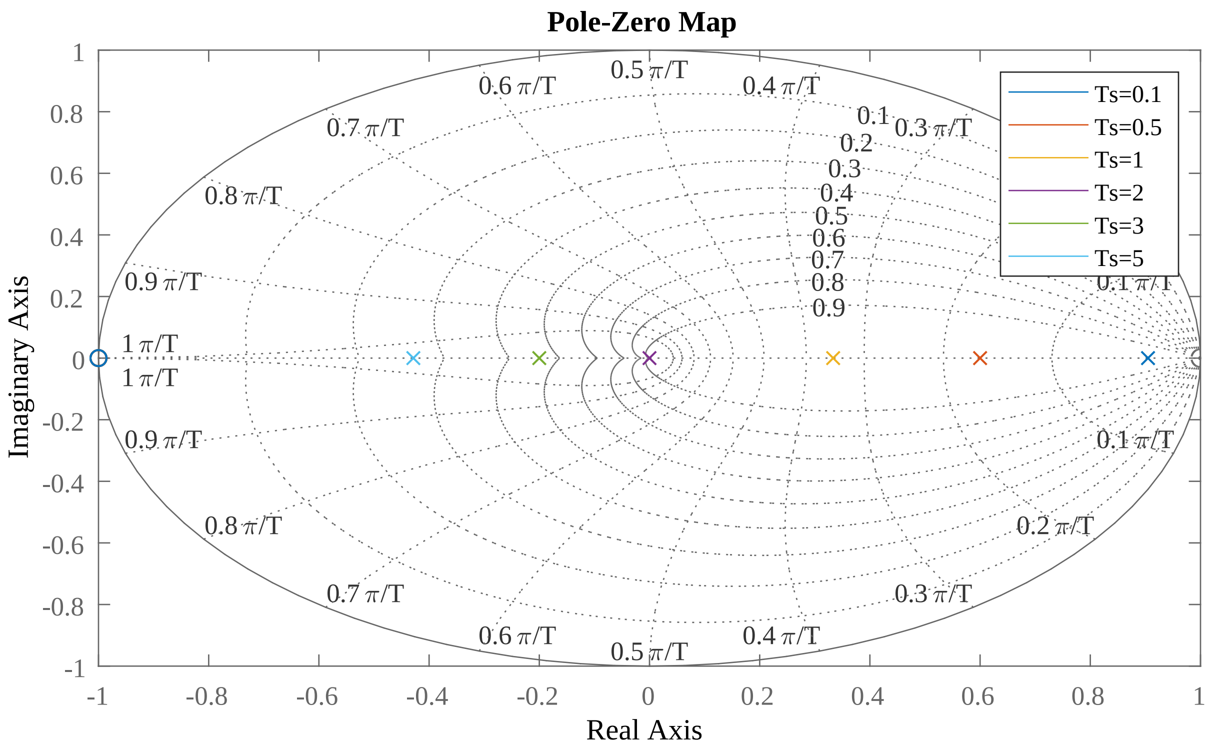
Lets consider the low pass filter below

$$G(s) = \frac{1}{s+1} \tag{27}$$

Convert the transfer function to the z-domain using forward euler approximation for s.

$$G(z) = \frac{T + Tz^{-1}}{(2+T) + z^{-1}(T-2)} \tag{28}$$


1.2
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8 9 10
Ts=0.1
Ts=0.5
Ts=1
Ts=2
Ts=3
Ts=5
cont


Pole-Zero Map
Imaginary Axis
Real Axis
Ts=0.1
Ts=0.5
Ts=1
Ts=2
Ts=3
Ts=5
0.5 π/T
0.6 π/T
0.4 π/T
0.7 π/T
0.3 π/T
0.8 π/T
0.9 π/T
1 π/T
0.1 π/T
0.2 π/T
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9