

# DIGITAL CONTROL OF POWER ELECTRONICS

## Intro to Continuous Control Part 1

We will be dealing with controllers, sensors, and plants in the Laplace domain. Recall the Laplace transform as

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} f(t)e^{-st} dt \quad (1)$$

A transfer function generally defined as a rational polynomial with a numerator and denominator.

$$G(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{k=0}^n b_k s^k}{\sum_{k=0}^m a_k s^k} \quad (2)$$

Below the transfer function is shown as a rational polynomial in the numerator and denominator. The roots of the numerator are called zeros of the transfer function because they render the transfer function zero and the roots of the denominator polynomial are called the poles.

$$G(s) = \frac{Y(s)}{X(s)} = \frac{a_n(s - z_1)(s - z_2) \dots (s - z_n)}{b_m(s - p_1)(s - p_2) \dots (s - p_m)} \quad (3)$$

The dashed lines of constant radius indicate lines of constant natural frequency and the dashed lines of constant angle represent lines of constant damping ratio. The angle of 180 degrees, real and negative, is a constant damping ratio 1.

$$G(s) = \frac{1}{s^2 + 2\alpha s + \omega_0^2} \quad (4)$$

$$G(s) = \frac{1}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \quad (5)$$

The neper frequency is  $\alpha$ , the natural frequency is  $\omega_0$ , damped radian frequency is  $\omega_d$ , and the damping ratio is  $\zeta$ .

$$p = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \quad (6)$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} \quad (7)$$

$$p = -\alpha \pm j\omega_d \quad (8)$$

$$\zeta = \frac{\alpha}{\omega_0} \quad (9)$$

$$p = -\zeta\omega_0 \pm j\omega_0\sqrt{1 - \zeta^2} \quad (10)$$

Consider the 5 systems below. Lets investigate their poles, zeros, step response, and bode plots.

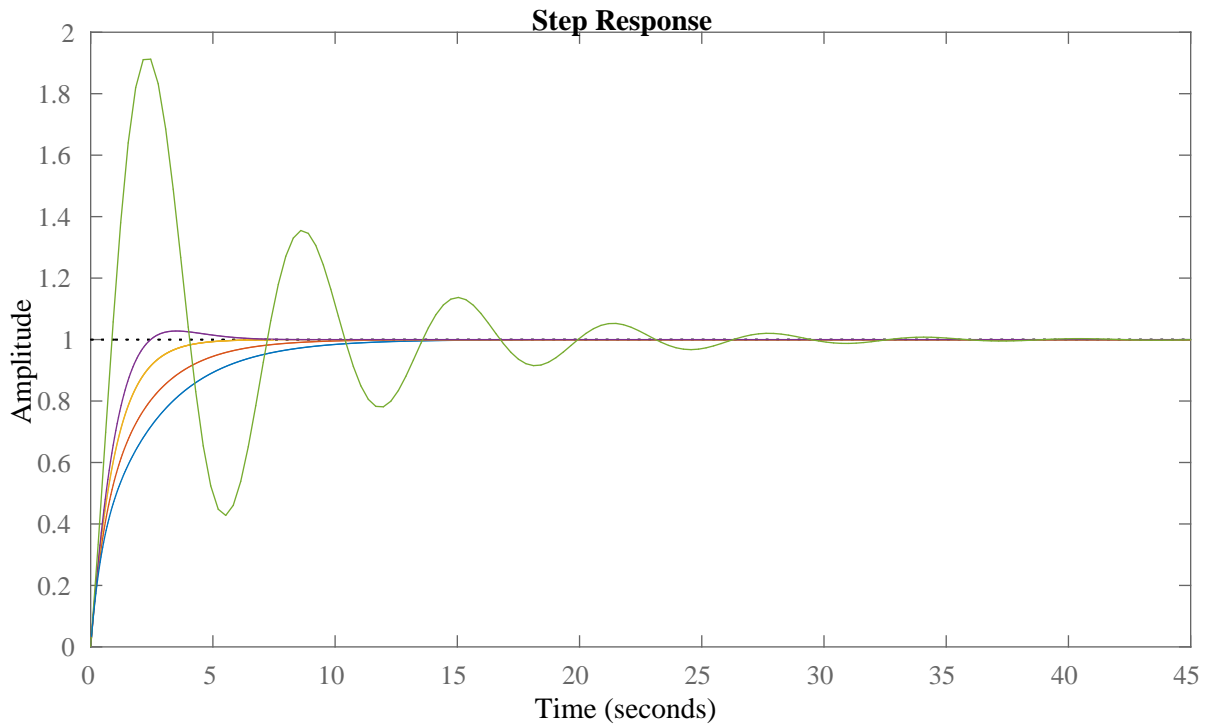
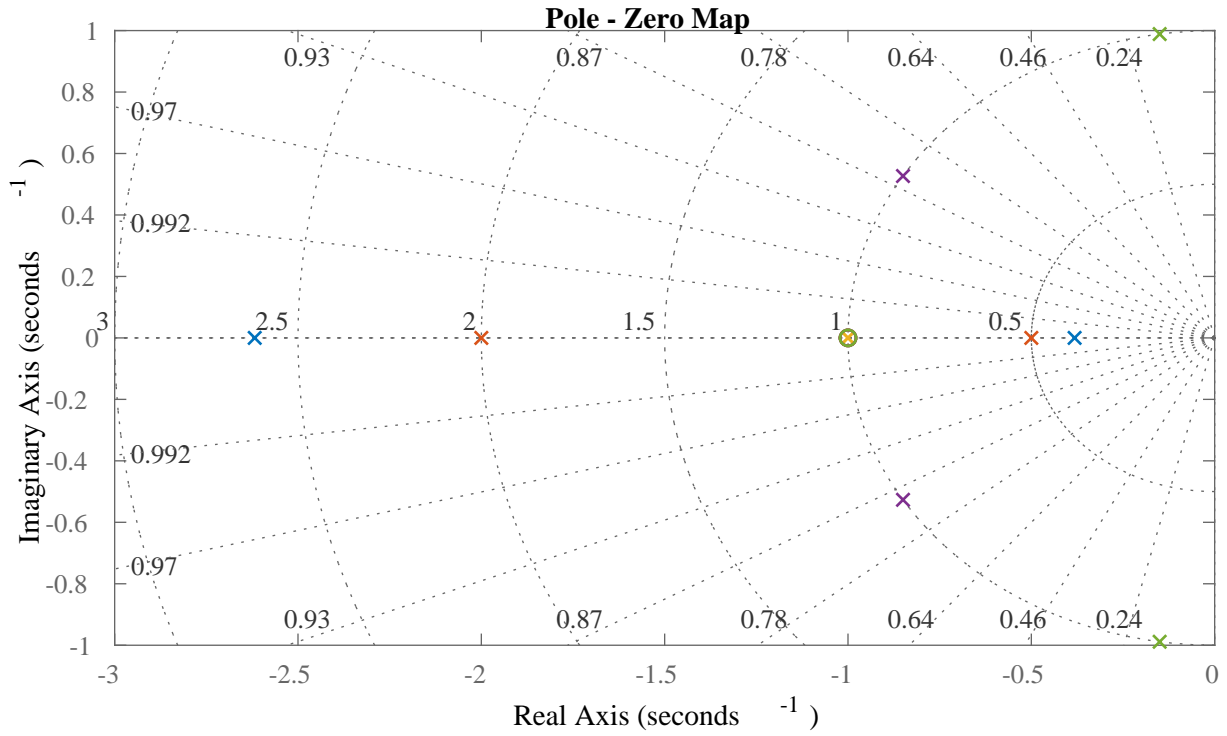
$$G(s) = \frac{s + 1}{s^2 + 3s + 1} \quad (11)$$

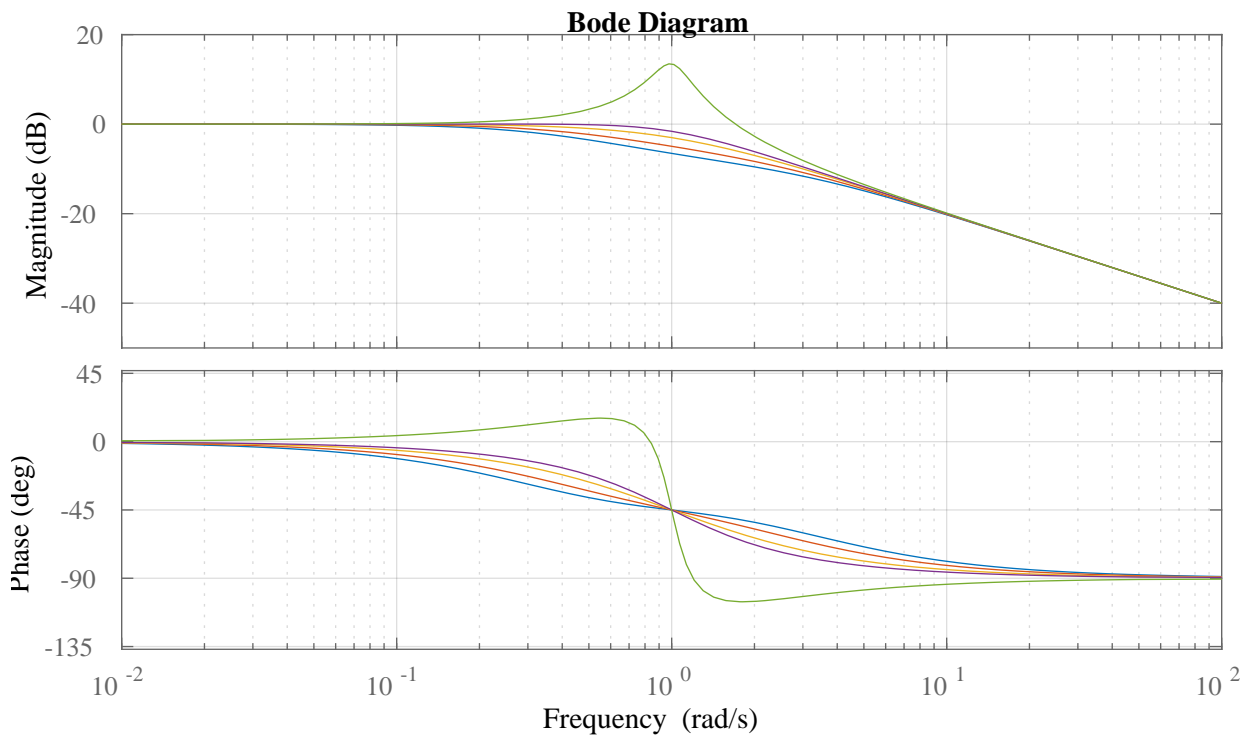
$$G(s) = \frac{s + 1}{s^2 + 2.5s + 1} \quad (12)$$

$$G(s) = \frac{s + 1}{s^2 + 2s + 1} \quad (13)$$

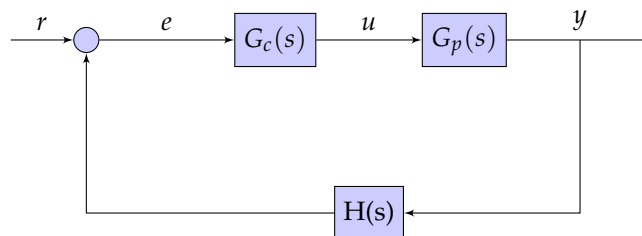
$$G(s) = \frac{s + 1}{s^2 + 1.7s + 1} \quad (14)$$

$$G(s) = \frac{s + 1}{s^2 + 0.3s + 1} \quad (15)$$





A general block diagram of a control systems is shown below for power electronics.



Plant:  $G_p(s)$  process trying to control

Controller:  $G_c(s)$  transfer function that stabilizes system and tracks the reference

Feedback:  $H(s)$  Sensor transfer function used for observing the quantity that you want to control

The open loop gain is defined as

$$G_{ol}(s) = G_c(s)G_p(s)H(s) \tag{16}$$

The reference to output transfer function is defined as

$$G_{cl}(s) = \frac{y(s)}{r(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)H(s)} \tag{17}$$

The reference to control effort transfer function is defined as

$$\frac{u(s)}{r(s)} = \frac{G_c(s)}{1 + G_c(s)G_p(s)H(s)} \tag{18}$$

The disturbance to output transfer function is defined as

$$\frac{y(s)}{d(s)} = \frac{1}{1 + G_c(s)G_p(s)H(s)} \tag{19}$$

The disturbance to control effort transfer function is defined as

$$\frac{u(s)}{d(s)} = \frac{G_c(s)H(s)}{1 + G_c(s)G_p(s)H(s)} \quad (20)$$

### Stable Control Loop

- Different methods to determine stability: Routh-Hurwitz criterion, Nyquist criterion, Bode diagram.
- Closed loop transfer function has all poles in left hand plane or in other words the closed loop transfer function has no poles in the right hand plane
- In the bode plot, if the open loop transfer function passes through the gain of 0dB before the phase of the open loop transfer reaches more than -180 degrees, the closed loop transfer function will be stable.
- The phase margin is defined as the amount of degrees that you are away from -180 degrees when the gain crosses through 0 db. In general, a phase margin of 60 degrees or more gives good results.
- Response of system is dominated by poles that are closer to the origin in the s-plane.
- The farther the dominant poles are to the left in the s-plane, the faster the system response will be and the greater the bandwidth will be.
- The farther the dominate poles are to the left, the more actuation/input is used to push system to desired objective. This traditional means more energy input.